

MATH 250: DAILY PREPARATION

Overview

To date, we have learned a fair bit of logic, some key information about quantifiers and statements, and have done a reasonable amount of work with proving conditional statements directly. As we embark on the rest of Chapter 3, we now investigate alternate approaches to proving conditional statements. One of the key ideas that undergirds all of our study is this: two statements that *look* different at first can, in fact, be *logically equivalent*. Knowing these logical equivalences is a key part of your “proof toolbox,” since having different options to consider increases the possible routes we can take to finding a valid proof.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- Know the *converse* and *contrapositive* of any conditional statement and which of these is logically equivalent to the original conditional statement.
- Know how to negate any conditional statement.
- Be able to execute the first steps of a standard direct proof of a conditional statement. That is, you should know what to assume at the outset and what your goal is to show.
- Understand the basic idea of *direct proof by contraposition* (this language is not used in the text, but it is in the first two videos).
- Know what a *biconditional* statement is and how it is logically equivalent to the conjunction of two conditional statements.

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform in the near future **with practice and further study**:

- Understand how to perform a direct proof by contraposition. That is, how to prove a conditional statement by instead proving that its contrapositive is true. You should know both how the logic of this approach works, as well as how we write the result formally in a proof.
- Understand how to prove a biconditional statement, including how it is possible to use direct proof by contraposition within either of the two “directions”.
- Know what we mean by the *forward* and *backward* directions of a conditional statement.

Resources

Reading: Read pages 102 - 105.

Watching: Here are some additional resources that have been developed to support your learning:

- Screencast 3.2.1: <http://gvsu.edu/s/rx>
- Screencast 3.2.2: <http://gvsu.edu/s/ry>
- Screencast 3.2.3: <http://gvsu.edu/s/rz>

Questions

Respond to the following questions on separate paper, as explained in the document that describes guidelines and expectations for daily preparatory assignments. You should be prepared to show me your responses at the start of class; I will review your work briefly sometime before the end of class.

1. Given a conditional statement $P \rightarrow Q$, what is the converse of the statement? the contrapositive?
2. Is the converse of $P \rightarrow Q$ logically equivalent to the original statement? Is the contrapositive of $P \rightarrow Q$ logically equivalent to the original statement?
3. Complete Preview Activity 1 in Section 3.2.
4. Complete Preview Activity 2 in Section 3.2.
5. What is one important lesson that you've learned from today's preview activities, reading, and watching?