MATH 250: DAILY PREPARATION

Overview

As we continue to develop different methods of proving statements in Chapter 3, the next approach we encounter is the powerful method of *proof by contradiction*. This approach rests on the law of the excluded middle: any statement S is either true or false, but not both. It also rests on a double negative: if we can show it cannot be the case that $\sim S$ is true, then S must be true. That is, the basic approach is to show "it is not the case that not-S is true." Obviously, there's a fair bit to keep track of in a proof by contradiction, so you'll want to watch the videos for today and read and think carefully about proof by contradiction. We'll devote the vast majority of class on Thursday to discussion of this method.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- Be able to comfortably negate any statement. Especially conditional statements.
- Understand the basic premise behind proof by contradiction. To prove statement S, we assume . . ., and strive to show that . . ..
- Know some of the introductory sentences we write to start any proof by contradiction (see the Writing Guidelines in the assigned reading).

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform in the near future **with practice and further study**:

- Recognize certain types of statements that naturally suggest proof by contradiction might be a fruitful line of reasoning.
- Be able to correctly execute proofs that use a contradiction argument.
- Realize the power of the tautology "P or $\sim P$."

Resources

Watching: For Thursday, I want you to watch some video before you read. In particular, watch the following two videos.

- Screencast 3.3.1: http://gvsu.edu/s/rB
- Screencast 3.3.2: http://gvsu.edu/s/rC

Reading: Read pages 118-120.

Questions

Respond to the following questions on separate paper, as explained in the document that describes guidelines and expectations for daily preparatory assignments. You should be prepared to show me your responses at the start of class; I will review your work briefly sometime before the end of class.

1. Negate this statement: "For all real numbers x and y, if $xy = 0$, then $x = 0$ or $y = 0$."

2. Suppose that I want to prove the statement $P \rightarrow Q$ using a contradiction argument. What do I get to assume?

3. If you are doing a pre-proof analysis for an argument that $P \rightarrow Q$ where you are planning to do a proof by contradiction, what should you write following "we want to show that ..."?

4. Complete Preview Activity 2 in Section 3.3. (You can either read or simply skip Preview Activity 1.)