MATH 250: DAILY PREPARATION

Overview

With Sections 3.1 - 3.3 (direct proofs, direct proof by contraposition, and proof by contradiction), we have encountered the three main ways that we can approach the proof of a conditional statement. In Section 3.4, we encounter an approach that can assist us in any of these three aforementioned approaches. In particular, we study the notion of "proof by cases," wherein we see how in certain settings, we can add structure to our assumptions without assuming any more than we've actually been given. For instance, if we are allowed to assume that n is an integer, it is certainly fair to say that our arbitrary integer n must be either even or odd. As long as we account for both cases, we are entirely within the bounds of valid reasoning. Such additional structure in our assumptions (getting to assume that $n = 2k$ or $n = 2k + 1$) will often be very helpful to us.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- Understand the basic structure of a proof that uses cases and how this relies upon a conditional statement whose hypothesis is a disjunction.
- Be able to recognize some common situations where cases can be used to add structure to our assumptions in a proof.
- Apply the basic writing guidelines for a proof by cases.

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform in the near future **with practice and further study**:

- Become comfortable applying case structures in different arguments in different settings (this will take time, maybe even over the remainder of the course).
- Be able to use congruence modulo n to generate a case structure in a proof that regards divisibility by n.

Resources

Reading: Read pages 131-135. You should read Preview Activity 1 in Section 3.4, but you need not answer the questions in writing.

Watching: Here are some additional resources that have been developed to support your learning:

- Screencast 3.4.1: http://gvsu.edu/s/s9
- Screencast 3.4.3: http://gvsu.edu/s/sa

Questions

Respond to the following questions on separate paper, as explained in the document that describes guidelines and expectations for daily preparatory assignments. You should be prepared to show me your responses at the start of class; I will review your work briefly sometime before the end of class.

1. Complete Preview Activity 2 on p. 132-133.

2. Complete Progress Check 3.21.

3. Why does a proof involving $|x|$ often suggest that we use cases? How many cases would we consider?

4. Consider the statement "For any integer n, if $n \not\equiv 1 \pmod 4$, then $n \not\equiv 0 \pmod 2$." If you were going to attempt to prove this statement directly, how many cases would there be to consider? What, specifically, are the cases?