MATH 250: DAILY PREPARATION

Overview

In Chapter 3, we studied the main structural approaches to proving conditional statements, and along the way we learned some important number theory involving divisibility and congruence. In our remaining work in the course, we will be focused on learning some new mathematical content while also learning additional proof techniques and perspective. Up first in Chapter 4 is the topic of *mathematical induction* which is an approach that is used for a class of theorems that make statements about all (or nearly all) natural numbers. A typical result in Chapter 4 will have form "*for all natural numbers n, $P(n)$ is true*." We will see how the very nature of the natural numbers and the notion of an *inductive set* leads us to be able to rigorously prove statements in this form.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- Know the definition of an inductive set.
- Be able to start a direct proof of any statement of the form "if $P(k)$ is true, then $P(k+1)$ is true."
- Understand the metaphor of "the magical gnome-infested staircase" (Screencast 4.1.1) and its relationship to the natural numbers.
- Know the Principle of Mathematical Induction and how we use it in a proof, including what we mean by the *basis step* and *inductive step*.

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform in the near future **with practice and further study**:

- Understand the notation $\sum_{i=1}^{n} f(n)$ and use this notation comfortably in induction proofs.
- Apply mathematical induction to several different types of statements and write proofs according to the Writing Guidelines for mathematical induction.
- Use correct and proper notation with predicates and variables to write valid induction proofs.

Resources

Reading: Read pages 169-174 (through the box at the top of p. 174), plus the short section "Summation Notation" at the bottom of p. 176 and top of p. 177.

Watching: Here are some additional resources that have been developed to support your learning:

- Screencast 4.1.1: `http://gvsu.edu/s/sz`
- Screencast 4.1.2: `http://gvsu.edu/s/sA`
- Screencast 4.1.3: `http://gvsu.edu/s/sB`

Continued →

Questions

Respond to the following questions on separate paper, as explained in the document that describes guidelines and expectations for daily preparatory assignments. You should be prepared to show me your responses at the start of class; I will review your work briefly sometime before the end of class.

1. Complete Preview Activity 1 in Section 4.1.
2. Please read Preview Activity 2; you need only write down an answer to question #2g in this Preview.
3. Write the statement of the Principle of Mathematical Induction, and then write (in your own words), an explanation of how we apply the Principle of Mathematical Induction to prove a statement of the form "for every natural number n, $P(n)$ is true."
4. Write the following sum in expanded form (that is, without using the symbol Σ): $\displaystyle\sum_{i=2}^{5} i^2$.
5. Write the following sum in shorthand form (that is, in Σ notation): $3 + 5 + 7 + 9 + 11 + 13$.