

MATH 250: DAILY PREPARATION

Overview

We have recently started Chapter 4, and the typical results we are considering have form “for all natural numbers n , $P(n)$ is true.” To prove a result in this form, we use the Principle of Mathematical Induction. Sometimes, however, a result may hold for nearly all natural numbers, but not every single one. If there are only a finite number of natural numbers for which $P(n)$ is false, then we can prove a related result of the form “for all natural numbers n such that $n \geq M$, $P(n)$ is true.” The key in this situation is to identify the constant m , which is the minimum value for which the predicate $P(n)$ holds. Basically, we adjust the basis step of the induction argument to start with $n = M$, rather than $n = 1$, and everything else stays the same in terms of the spirit and process of the argument. As such, there are not many new ideas in Section 4.2, though our work in this part of the text will provide substantial opportunities for continuing to get better at induction proofs.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- State the Principle of Mathematical Induction. Understand how the Extended Principle of Mathematical Induction is closely related to the original Principle.
- Understand the ideas of the basis step and the inductive step in a proof by mathematical induction.

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform in the near future **with practice and further study**:

- Understand how to apply inductive arguments in a wide range of mathematical settings, including to prove inequalities, formulas for derivatives, statements regarding divisibility and congruence, and more.
- Use correct and proper notation with predicates and variables to write valid induction proofs.

Resources

Reading: Read pages 188-192, up to Progress Check 4.8.

Watching: Here are some additional resources that have been developed to support your learning:

- Screencast 4.2.1: <http://gvsu.edu/s/tg>

Questions

Respond to the following questions on separate paper, as explained in the document that describes guidelines and expectations for daily preparatory assignments. You should be prepared to show me your responses at the start of class; I will review your work briefly sometime before the end of class.

1. What is the main difference between the Principle of Mathematical Induction and the *Extended* Principle of Mathematical Induction?
2. Complete Preview Activity 1 in Section 4.2.
3. In Preview Activity 1, what would be the “basis step” in the argument? That is, what n -value is used and what statement is proved in this basis step?
4. Is the inductive step in a proof using the Extended Principle of Mathematical Induction exactly the same as in using the original Principle of Mathematical Induction, or is there something different? Explain, by clearly stating what the inductive step is in an inductive proof that will ultimately demonstrate that for all “ $n \geq M$, $P(n)$ is true.”