Math 311 - Homework 1

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1. (a) Let a natural number n be expressed in base 10 as

$$n = a_k a_{k-1} \cdots a_1 a_0.$$

   (Note that what we mean by this notation is that each a_i is a diigit of a regular base 10 number, not that the a_i are being multiplied together.) If $m = a_k + a_{k-1} + ... + a_1 + a_0$ then $n \equiv m \mod 3$.

   (b) Using what you showed in (a), prove that a natural number that is expressed in base 10 is divisible by 3 if and only if the sum of its digits is divisible by 3.

2. Suppose that the sequence $\{a_n\}$ of numbers defined by $a_1 = 1$, $a_2 = 2$, $a_3 = 3$ and

$$a_n = a_{n-1} + a_{n-2} + a_{n-3}$$

   for all $n \geq 4$. Show that $a_n < 2^n$.

3. Use the Division Algorithm to establish the following:
   (a) The square of any integer is either of the form $3k$ or $3k + 1$.
   (b) The fourth power of any integer is either of the form $5k$ or $5k + 1$.
   (c) $3a^2 - 1$ is never a perfect square for any integer a.

4. (a) Prove that the product of any three consecutive integers is divisible by 6.
   (b) Prove that the expression $(3n)!/(3!)^n$ is always an integer for every $n \geq 1$.