
HOMework #3

- (a) The product of two or more numbers of the form $4k + 1$, is also of that form.
(b) There are infinitely many prime numbers of the form $4k + 3$.
- Suppose $\gcd(a, b) = 1$. For any inter $k > 0$, prove that the arithmetic progression

$$a + b, a + 2b, \dots$$

contains k consecutive terms that are composite. [Hint: find a special integer n so that $a + (n + 1)b, \dots, a + (n + k)b$ are all composite].

- When eggs in a basket are removed two, three, four, five or six at a time, there remain, respectively, one, two, three, four, or five eggs. When they are taken out seven at a time, none are left over. Find the smallest number of eggs that could have been contained in the basket.
- Given any positive integer k , prove that there are k consecutive integers each divisible by a square > 1 . [Hint: Chinese Remainder Theorem!]