HOMEWORK #4

1. Let p be an odd prime. Prove that

   (a) $1^{p-1} + 2^{p-1} + ... + (p-1)^{p-1} \equiv -1 \mod p$

   (b) $1^p + 2^p + ... + (p-1)^p \equiv 0 \mod p$

2. If m and n are relatively prime positive integers, prove that

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \mod mn.$$

3. Find all positive integers n such that

   (a) $\phi(n) = 16$

   (b) $\frac{\phi(n)}{n} = \frac{2}{3}$

4. For natural numbers a, b and n, suppose that $ord_n(a) = h$ and $ord_n(b) = k$. Show that the $ord_n(ab)$ divides hk. Use this to show that if $\gcd(h, k) = 1$ then $ord_n(ab) = hk$.