
HOMEWORK #5

1. if p is an odd prime, prove the following:
 - (a) The only incongruent solutions to $x^2 \equiv 1 \pmod{p}$ are 1 and $p - 1$.
 - (b) The congruence
$$x^{p-2} + x^{p-3} + \dots + x^2 + x + 1 \equiv 0 \pmod{p}$$
has exactly $p - 2$ incongruent solutions, they are the integers $2, 3, \dots, p - 1$.
2. Assuming that r is a primitive root of the odd prime p , establish the following facts:
 - (a) The congruence $r^{\frac{p-1}{2}} \equiv -1 \pmod{p}$ holds.
 - (b) If r' is any other primitive root modulo p , then rr' is not a primitive root modulo p .
(*Hint: By part (a), $(rr')^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.*)
 - (c) If the integer r' is such that $rr' \equiv 1 \pmod{p}$, then r' is a primitive root of p .
3. Find all positive integers less than 61 having order 4 modulo 61. Explain your work in a way that I can follow.
4. There will be a seminar talk on Monday November 5 at 2:00-2:50 in College Hall 446. Go to this talk, and write down three meaningful facts and one interesting question that demonstrates that you gave some thought to the presentation (for example "Why did you decide to study this?" doesn't count). If you can't be at the talk for work reasons, let me know and I have an alternative for you.