HOMEWORK #2

1. Confirm the following properties of the greatest common divisor:

   (a) If $\gcd(a, b) = \gcd(a, c) = 1$, then $\gcd(a, bc) = 1$. *(Hint: use the hypothesis to show that 1 can be written as $ak + (bc)m$ for some integers k and m.)*

   (b) If $\gcd(a, b) = 1$ and $c \mid (a + b)$ then $\gcd(a, c) = \gcd(b, c) = 1$.

2. For integers a and b suppose that x' and y' are solutions to the diophantine equation

$$ax + by = d.$$

   (a) Show that

$$x = x' + \frac{b}{\gcd(a, b)}t \text{ and } y = y' - \frac{a}{\gcd(a, b)}t$$

   is also a solution for any $t \in \mathbb{Z}$.

   (b) Suppose that x_0 and y_0 are also solutions to the diophantine equation $ax + by = d$. Show that x_0 and y_0 have the form give in part (a). *(Hint: first, notice that we can always write $x_0 = x' + k$ and $y_0 = y' + m$ for integers k and m. That's just basic arithmetic. Now show that*

$$k = \frac{b}{\gcd(a, b)}t \text{ and } m = -\frac{a}{\gcd(a, b)}t$$

   for some integer t)

3. For the Diophantine equation $123x + 360y = 99$ determine

   (a) all its solutions;

   (b) all is solutions in positive integers.

4. (a) For $n > 2$, show that every prime divisor of $n! - 1$ is greater than n.

   (b) Prove that if $n > 2$, then there exists a prime p satisfying $n < p < n!$.