

1. Mersenne Primes, What are Mersenne primes? What is the connection between perfect numbers and Mersenne Primes? What is the largest known Mersenne prime? Is there a prize for finding the next largest one?
2. Apollonian Circle packings. Explain Apollonius' Theorem, and explain how this theorem can be used to generate a circle packing. Highlight its connection to number theory (hint: the reciprocal of the radii are solutions to some diophantine equation). I have some resources for you if you choose this one.
3. The Pell equation. Give a brief history of Pell's equation. If you are able to find the one fundamental solution, then it is a very old result of Bhramagupta that you can find infinitely many solutions. If you are feeling ambition, you can also explain how to find the fundamental solution using a very cool method called continued fractions.
4. Pseudoprimes and Carmichael numbers.
5. Applications of number theory to bitcoin and the blockchain. I have a few articles and blogposts that might steer you in the right direction.
6. Partition functions. What is a partition function? Give some examples of well known congruences involving partition functions. I also have a Scientific American article about Ramanujan and Ken Ono that will be helpful.
7. Coding Theory and Sphere Packings. I can give you an article about sphere packing and coding theory that appeared in Scientific American a few years ago. In this talk, you should be sure to talk about the different types of packings and which are the most efficient.
8. Graham's Number. This is often called "the largest number that anyone would ever take seriously." Where does it come from, and what is it good for (or not good for, if you prefer) (<https://www.youtube.com/watch?v=HX8bihEe3nA>)? How big is it (<https://www.youtube.com/watch?v=Guigptw1VHo>)?
9. Fibonacci Numbers. Explain how to generate the Fibonacci sequence recursively. Describe some of the interesting identities involving Fibonacci numbers. If you have time, you can also talk about Lucas numbers, a relative of the Fibonacci numbers.
10. Fermat's Theorem on sums of two squares. What are some examples of things that can be expressed as sums of two squares? What about things that definitely can't be? If time permits, this can also be extended to include Legendre's three-square theorem and Lagrange's four-square theorem.
11. Triangular numbers. In 1796 Gauss noticed that every natural number can be written as a sum of three triangular numbers. Explain what triangular numbers are, and give us

a proof of Gauss' theorem. (Hint: the proof will involve Legendre's theorem on sums of three squares.)

12. Number theory and the calendar. Show how we can use number theoretic functions to give the day of the week for any date in history. A bonus of this project, is that if you learn the algorithm (it's not too bad once you learn it) you can do crazy calculations and really impress your friends. If time permits, you can also look into John Conway's Doomsday algorithm.
13. Biorhythms: this is the idea that our body works on several different cycles of variable lengths and we can use the Chinese Remainder Theorem to optimize our performance in life.
14. Any other topic that you think looks interesting!